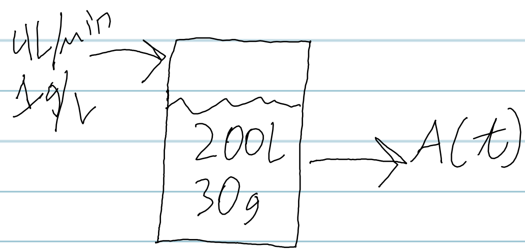
3.1: Applications

Ex) pg 91, #21 A tank contains 200L of fluid in which 30g of salt is disolved. Brine containing 1 gram of salt per litre is pumped into the tank at a rate of 4L/min. The well-mixed solution is pumped out at the same rate. Find the number A(t) of grams of salt in the tank at time t.



Initial Condition

$A_0 = 200L$ with $30g$

Brine $4g/min$ $(4L/min \mid 1g/L)$

Input Rate $= 4L/min$

Output Rate $= 4L/min$

goal/ $A(t)$ = amount of salt at time (t)

we need a D.E.

$$\frac{dA}{dt} = \text{rate}_{in} - \text{rate}_{out}$$

$$= (\text{consentration}_{in})(\text{flow rate}_{in}) - (\text{concentration}_{out})(\text{flow rate}_{out})$$

$$= (1g/L)(4L/min) - (\,?\,)(4L/min)$$

$$\frac{da}{dt} = \left(\frac{1\text{ gram}}{L}\right)\left(4\,L/\text{min}\right) - \left(\frac{a(t)}{200\,L}\right)\left(4\,L/\text{min}\right)$$

$$\frac{da}{dt} = \left(\frac{4\text{ grams}}{\text{min}}\right) - \left(\frac{A\text{ grams}}{50\text{ min}}\right)$$

$$\mu(t) = e^{\int \frac{1}{50}dt} = e^{\frac{1}{50}t}$$

$$= 4 - a/50$$

$$\frac{da}{dt} + \frac{1}{50}a = 4$$

$$e^{\frac{1}{50}t}\frac{da}{dt} + e^{\frac{1}{50}t}\cdot\frac{1}{50}a = 4e^{\frac{1}{50}t}$$

$$\frac{d}{dt}\left[e^{\frac{1}{50}t}a\right] = 4e^{\frac{1}{50}t}$$

$$e^{\frac{1}{50}t}a = \int 4e^{\frac{1}{50}t}\,dt$$

$$e^{\frac{1}{50}t}a = 4(50)e^{\frac{1}{50}t} + C$$

$$= 200e^{\frac{1}{50}t} + C$$

$$a = 200 + \frac{C}{e^{\frac{1}{50}t}} \leftarrow \qquad a_0 = 30$$

$$a(0) = 200 + Ce^{-\frac{1}{50}(0)} = 30$$

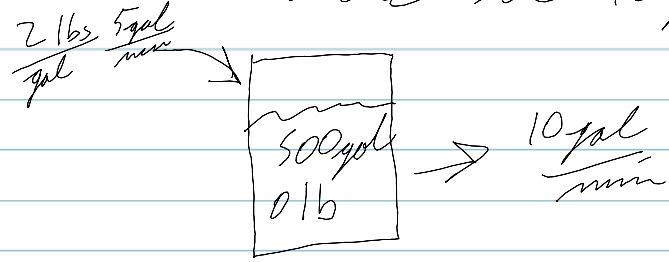
$$200 + C = 30$$

$$C = -170$$

$$\boxed{a(t) = 200 - \frac{170}{e^{\frac{1}{50}t}}}$$

Ex) 23 A large tank is filled to capacity with 500 gallons of pure water. Brine containing 2 lbs of salt per gallon is pumped into the tank at a rate of 5 gallons per minute. The well mixed solution is pumped out at the same rate. Find the number $A(t)$ of lbs of salt in the tank at time t.

#25 let out rate be 10 gal/min



$$\frac{dA}{dt} = \left(\text{concentration in}\right)\left(\text{flow rate in}\right) - \left(\text{concentration out}\right)\left(\text{flow rate out}\right)$$

$$= \left(\frac{2\text{ lbs}}{\text{gal}}\right)\left(\frac{5\text{ gal}}{\text{min}}\right) - \left(\quad\right)\left(\frac{10\text{ gal}}{\text{min}}\right)$$

$$\text{concentration out} = \frac{\text{lb}}{\text{gal}} = \frac{A}{500-5t}$$

$$\frac{dA}{dt} = \left(\frac{10\text{ lbs}}{\text{min}}\right) - \frac{2A}{100-t}\left(\frac{\text{lbs}}{\text{min}}\right)$$

$$\frac{dA}{dt} = 10 - \frac{2A}{100-t}$$

$$\frac{dA}{dt} + \frac{2}{100-t}A = 10$$

$$\mu(t) = e^{\int \frac{2}{100-t}dt}$$

$$= e^{-2\ln|100-t|}$$

$$= (100-t)^{-2} = \frac{1}{(100-t)^2}$$

$$\underbrace{\frac{1}{(100-t)^2}\frac{da}{dt} + \frac{1}{(100-t)^2}\left(\frac{2}{100-t}\right)a}_{} = \frac{1}{(100-t)^2}\cdot 10$$

$$\frac{d}{dt}\left[\frac{1}{(100-t)^2}A\right] = \frac{10}{(100-t)^2}$$

$$\frac{1}{(100-t)^2}A = 10\int (100-t)^{-2}\,dt$$

$$\frac{1}{(100-t)^2}a = 10\,\frac{(100-t)^{-1}}{-1}\cdot -1 + C$$

$$\frac{1}{(100-t)^2}a = \frac{10}{100-t} + C$$

$$a = 10(100-t) + C(100-t)^2 \qquad a_0 = 0$$

$$a_0 = 10(100-0) + C(100-0)^2 = 0$$
$$= 1000 + C(10{,}000) = 0$$
$$C(10{,}000) = -1000$$
$$C = -0.1$$

$$\boxed{a(t) = 10(100-t) - 0.1(100-t)^2}$$